WS 10 — Tangents, Normals, & Higher Order Derivs.

1) $y = 2x^3 + 3x^2 - 12x + 1$

$y' = 6x^2 + 6x - 12 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\boxed{x = -2,\ 1}$

2) $f(x) = (x^2+1)(x+3) = x^3 + 3x^2 + x + 3$

$f'(x) = 3x^2 + 6x + 1 = 0$

$x = \dfrac{-6 \pm \sqrt{36 - 12}}{6}$

$x = \dfrac{-6 \pm \sqrt{24}}{6} = -\dfrac{6}{6} \pm \dfrac{2\sqrt{6}}{6}$

$\boxed{x = -1 \pm \dfrac{\sqrt{6}}{3}}$

3) $y = x^2(3-x) = 3x^2 - x^3$

point: $(-2, 20)$

slope: $y' = 6x - 3x^2$, $y'(-2) = -24$

Tangent: $y - 20 = -24(x+2)$

Normal: $y - 20 = \frac{1}{24}(x+2)$

4) $f(x) = \sqrt{x}$ $\quad x = 4$

point: $(4, 2)$

slope: $f'(x) = \dfrac{1}{2\sqrt{x}}$, $f'(4) = \frac{1}{4}$

Tangent: $y - 2 = \frac{1}{4}(x-4)$

Normal: $y - 2 = -4(x-4)$

5) $y = x\sqrt{x} = x^{3/2}$; $\quad y = 1 + 3x$

slope: $y' = 3$

point: $y' = \frac{3}{2}\sqrt{x}$

$\frac{3}{2}\sqrt{x} = 3$

$\sqrt{x} = 2$

$x = 4$

$(4, 8)$

$\boxed{y - 8 = 3(x-4)}$

6) $y = x^3 - 3x + 1$ $\quad (2, 3)$

$y' = 3x^2 - 3$

$y'(2) = 9$

Normal: $\boxed{y - 3 = -\frac{1}{9}(x-2)}$

7) $y = (x^3 - 3x + 1)(x+2)$ $\quad x = 1$

point: $(1, -3)$

slope: $y' = (x^3 - 3x + 1) + (x+2)(3x^2 - 3)$, $y'(1) = -1$

$y + 3 = -1(x-1)$

8) $y = 2x^3 - 3x^2 - 12x + 20$

$y' = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$(-1, 27) \quad (2, -24)$

9) $f(x) = \frac{1}{x}$

$f'(x) = -x^{-2}$

$f''(x) = 2x^{-3}$

10) $y = \sqrt{x}$

$y' = \frac{1}{2\sqrt{x}}$

$y'' = -\frac{1}{4}x^{-3/2}$

11) $h(x) = f(x) + g(x)$

$h'(x) = f'(x) + g'(x)$

$h'(2) = f'(2) + g'(2) = -1 + \frac{3}{2} = \boxed{\frac{1}{2}}$

12) $d(x) = f(x) - g(x)$

$d'(x) = f'(x) - g'(x)$

$d'(3) = f'(3) - g'(3)$

$= \boxed{-2}$

13) $p(x) = f(x) \cdot g(x)$

$p'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

$p'(4) = f(4) \cdot g'(4) + g(4) \cdot f'(4)$

$= (2)(1) + (5)(-1)$

$= \boxed{-3}$

14) $q(x) = \frac{f(x)}{g(x)}$

$q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$q'(2) = \frac{g(2) \cdot f'(2) - f(2)g'(2)}{[g(2)]^2}$

$q'(2) = \frac{(3)(-1) - (4)(\frac{3}{2})}{9}$

$= \frac{-3-6}{9} = \boxed{-1}$

15) $m(x) = f(g(x))$

$m'(x) = f'(g(x)) \cdot g'(x)$

$m'(6) = f'(g(6)) \cdot g'(6)$

$= f'(4) \cdot (-2)$

$= \boxed{2}$